Multi-Stage Revenue Management with Inter-Temporal Dependence

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This example is adapted (almost verbatim) from test problem 2 by Prof. J.M. Harrison for class OIT 603 at Stanford University.

A businessman chooses to buy $b > 0$ units of capacity, paying $c > 0$ dollars per unit of capacity at $t = 0$. During stage t ($t = 1, \ldots, T$) he observes demand D_t for units at price p_t, at which point, he must choose to sell x_t units ($0 \leq x_t \leq D_t$), provided that the total number of units sold (accross all past periods) does not exceed b.

To better understand the initial capacity and selling decisions, consider the following: There is a large initial capacity b (units purchased). In period 1 the businessman receives a certain demand for units at low price p_{low}. The decision at period 1 is, esentially, choosing a number of units to be sold right now in order to reserve some units for customers arriving later because they will pay more for the product; The opportunity cost for each unit of capacity reserved is now p_{low}. Thus, in order to make a wise decision at period 1, the businessman must consider later demand.

Assume that $D_t = \mu_t X Y_t$ where X has a gamma distribution with parameters $k > 0$ and $\theta > 0$ such that it has mean $k\theta = 1$ and standard deviation $\sqrt{k}\theta = 1/\sqrt{k}$. $Y_1, \ldots, Y_T$ are i.i.d. exponential with mean 1 and μ_t are positive constants ($\forall\, t$).

Our goal is to calculate how many units should purchased (b) and how many units should be reserved for future periods in order to maximize total revenue. In other words, we want to find b and $r_t, t = 2, \ldots, T$ so that, if the number of units sold in all periods before t is less than $b - r_t$ (r_t units are reserved for periods $t, t+1, \ldots, T$), revenue is maximized.

Recommended Parameters: Take $c = \$80$, $T = 3$, $k = \theta = 1$ and μ_t, p_t as follows:

Period	1	2	3
Price (p_t)	100	300	400
Mean Demand μ_t	50	20	30

Starting Solutions: $b = 100$, $r_2 = 50$, $r_3 = 30$. If multiple solutions are needed, use $r_2 \sim$ Uniform(40,60) and $r_3 \sim$ Uniform(20,40).

Measurement of Time: Number of periods

Optimal Solution: Unknown