Vehicle Routing Problems with Stochastic Demands and Travel Times

Yao Luo and Eunji Lim

December 4, 2010

{Categorical variables, constrained.}


Problem Statement: Consider the Vehicle Routing Problem (VRP) defined on a graph \( G = (V, A) \), where \( V = \{v_0, v_1, v_2, \ldots, v_n\} \) is a set of vertices and \( A = \{(v_i, v_j) : i \neq j, v_i, v_j \in V\} \) is a set of arcs. Vertex \( v_0 \) denotes a depot where \( m \) identical vehicles are based, while the remaining vertices correspond to \( n \) customers. A matrix \( C = (c_{ij}) \) is defined on \( A \) with \( c_{ij} \) representing the distance between \( v_i \) and \( v_j \). A travel time matrix \( T = (T_{ij}) \) is also defined on \( A \) with \( T_{ij} \) denoting the random travel time from \( v_i \) to \( v_j \). A vector \( D = (D_1, D_2, \ldots, D_n) \) represents the random demands from the customers \( 1, \ldots, n \). Each vehicle, having the same capacity \( Q \), will start from the depot, visit a subset of the customers, and return to the depot. Service levels are measured in two ways. First, we evaluate the probability of the total demand along each route not exceeding the capacity \( Q \). Second, we evaluate the probability of the travel time along each route not exceeding some time limit \( B \). Our goal is to find the set of routes for the vehicles minimizing the total distance traveled by the vehicles while maintaining desired service levels. \( \theta = (\theta_1, \ldots, \theta_m) \) denotes the routes for the vehicles \( 1, \ldots, m \), where \( \theta_k \subset A \) and is the set of arcs for vehicle \( k \) to travel along \((1 \leq k \leq m)\). The problem can then be formulated as follows:

\[
\begin{align*}
\min & \quad \text{Total distance traveled by the vehicles} \\
\text{s.t.} & \quad P\left( \sum_{(v_i, v_j) \in \theta_k} D_j \leq Q \right) \geq \alpha_k, \quad 1 \leq k \leq m, \\
& \quad P\left( \sum_{(v_i, v_j) \in \theta_k, j \neq 0} T_{ij} \leq B \right) \geq \beta_k, \quad 1 \leq k \leq m,
\end{align*}
\]

or equivalently,

\[
\begin{align*}
\min & \quad \sum_{k=1}^{m} \sum_{(v_i, v_j) \in \theta_k} c_{ij} \\
\text{s.t.} & \quad P\left( \sum_{(v_i, v_j) \in \theta_k} D_j \leq Q \right) \geq \alpha_k, \quad 1 \leq k \leq m, \\
& \quad P\left( \sum_{(v_i, v_j) \in \theta_k} T_{ij} \leq B \right) \geq \beta_k, \quad 1 \leq k \leq m,
\end{align*}
\]

where \( \alpha_k \) and \( \beta_k \), \( 1 \leq k \leq m \), are the desired service levels.
Recommended Parameter Settings: \( n = m = 5, \alpha_k = \beta_k = 0.9 \) for \( 1 \leq k \leq m \), \( Q = 350 \), \( B = 240 \), and

\[
C = \begin{pmatrix}
0 & 35 & 78 & 76 & 98 & 55 \\
35 & 0 & 60 & 59 & 91 & 81 \\
78 & 60 & 0 & 3 & 37 & 87 \\
76 & 59 & 3 & 0 & 36 & 83 \\
98 & 91 & 37 & 36 & 0 & 84 \\
55 & 81 & 87 & 83 & 84 & 0
\end{pmatrix}.
\]

\( T_{ij} \) is uniformly distributed between \( 0.5c_{ij} \) and \( 1.5c_{ij} \) for \( 0 \leq i, j \leq n \). \( D_i \) is uniformly distributed between 110 and 190. The \( T_{ij} \)'s and \( D_i \)'s are independent.

Starting Solutions: Each vehicle serves exactly one customer, i.e., each vehicle starts at the depot, visits exactly one customer, and returns to the depot.

Measurement of Time: The use of one set of travel times and demands.

Optimal Solutions: Unknown.

Known Structure: None.

References