Restaurant Table Management

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(Real variables, constrained.)

Problem Description   Floor space in a restaurant is allocated to N different sizes of tables, with capacities $c = [c_1, c_2, ..., c_N], c_i \in Z_+^n$. A table of capacity c_i can seat c_i or less customers. The restaurant can seat a maximum of K customers at a time, regardless of table allocation, and is open for T consecutive hours. The decision variable is the vector $x \in (Z_+ \cup \{0\})^N$, the number of each size of table to use.

Groups of customers of size $j \in \{1, ..., \max_i(c_i)\}$ arrive according to a homogeneous Poisson process with rate λ_j. A group of customers is seated at the smallest possible available table. If there are no available tables large enough, the group of customers leaves immediately. Service time per group is exponential with means $s = [s_1, s_2, ..., s_{\max_i(c_i)}]$, and revenue per group is fixed at $r = [r_1, r_2, ..., r_{\max_i(c_i)}]$.

Let $N_j(t)$ be the number of groups of size j that were seated by time t with $N(t) = [N_1(t), ..., N_{\max_i(c_i)}(t)]$. The optimization problem is thus to maximize the total expected revenue:

$$\text{maximize} \quad N(T)r' \tag{1}$$

$$\text{subject to} \quad xc' \leq K \tag{2}$$

Recommended Parameter Settings: $T = 180 \text{minutes}, K = 80$
$c = (2, 4, 6, 8)$
$\lambda = (3, 6, 3, 3, 2, \frac{4}{3}, \frac{6}{5}, 1) * 60$
$s = (20, 25, 30, 35, 40, 45, 50, 60)$
$r = (15, 30, 45, 60, 75, 90, 105, 120)$

Starting Solution(s):   For one starting solution, distribute total capacity equally across table sizes, then distribute remaining capacity by descending table size. Eg, for the above settings, $\frac{80}{4} = 20$ capacity is assigned to each size, and the remaining 6 capacity is portioned to a table of size 6: $x = (10, 5, 4, 2)$

For multiple starting solutions, distribute total capacity uniformly across table sizes, then distribute remaining capacity by descending table size.

Measurement of Time: Number of replications of length T.

Optimal Solutions: Unknown.

Known Structure: Unknown.