Newsvendor Problem with Exogenous Stochastic Price

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(Integer-ordered variables, constrained.)

Problem Description Iron ore is traded on the spot market, facing an exogenous, stochastic price. There is enormous demand for iron, and so for the purposes of a small- or medium-sized iron ore mine, we assume that any quantity of ore can be instantaneously sold at current market rates.

Let there be T time periods (days), holding cost of h per unit, production cost of c per unit, maximum production per day of m units, and maximum holding capacity of K units. Let the iron ore market price for the day be P_t.

Let the decision variables be $x = [x_1, x_2, x_3, x_4]$, where x_1 is the price at which to begin production, x_2 is the inventory level at which to cease production, x_3 is the price at which to cease production, and x_4 is the price at which to sell all current stock.

The order of operations in the simulation is as follows:

1. Sample the market price, P_t. Let current stock be s_t.
2. If production is already underway,
   (a) if $P_t \leq x_3$ or $s_t \geq x_2$, cease production.
   (b) else, produce $\min(m, K - s_t)$ at cost c per unit.
3. If production is not currently underway, and if $P_t \geq x_1$ and $s_t < x_2$, begin production.
4. If $P_t \geq x_4$, sell all stock (after production) at price P_t.
5. Charge a holding cost of h per unit (after production and sales).

The optimization problem is to maximize total revenue over the T time periods.

Recommended Parameter Settings: $T = 1000, h = 1, c = 100, p = 100, K = 1000$. Let P_t be a mean-reverting random walk, such that $P_t = \text{trunc}(P_{t-1} + N_t(\mu_t, \sigma))$, where N_t are normal random variables with standard deviation $\sigma = 7.5$ and mean $\mu_t = \text{sgn}(\mu_0 - P_{t-1}) * |\mu_0 - P_{t-1}|^{1/4}$, $\mu_0 = 100$. $\text{trunc}(x) = \max(\min(x, 200), 0)$ (trunc bounds its argument in [0,200]). Let $P_1 = \mu_0$.

Starting Solution(s): $x = [80, 7000, 40, 100]$.

If multiple starting solutions are required, sample x_1, x_3, x_4 from $U(70, 90), U(30, 50), U(90, 110)$ respectively, and sample x_2 from the discrete uniform distribution [2000,8000].

Measurement of Time: Number of simulation replications of length T.

Optimal Solutions: Unknown.

Known Structure: Unknown.