Surgery Planning Problem

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(Binary vector variables, constrained.)
This example is adapted from the article by Lamiri el al. [**?**]

Problem Statement:
Consider the operating rooms (ORs) in a hospital. ORs are used to serve two types of patients: elective and emergency patients. Elective patients spend time on a waiting list before being given an appointment for their surgery, while emergency patients arrive randomly and require immediate surgical interventions, i.e., they should be operated as soon as possible on the arrival day. Assume that ORs are identically equipped, that each patient can be assigned to any OR, and that only the total available capacity of all ORs are considered.

For a finite planning horizon of H periods (days) and a set of N elective patients waiting to be operated, we want to determine the set of elective patients to be operated in each period over the planning horizon. Let T_t be the total ORs' available regular capacity in period t for $t = 1, 2, \ldots, H$, i.e., the number of regular working hours on the period t. If the duration of planned surgeries and emergency surgery exceeds the regular capacity, overtime costs c_t are incurred on period t. Let W_t be the (random) total capacity needed for emergency cases arriving on period t, and $f_{W_t}(\cdot)$ be the the density function of W_t. Assume that elective patients can be safely delayed and planned for a future date, i.e., they can be planned in advance. Let d_i be the operating time, and r_i the release period for $i \in \{1, 2, \ldots, N\}$. The release period represents the earliest period that elective i can be performed. Also let p_{it} be the cost of performing elective case i in period t for $i \in \{1, 2, \ldots, N\}$, $t = \{r_i, \ldots, H, H+1\}$. Then we want to efficiently assign elective patients to different periods in order to minimize the total costs of both elective patients' assignment costs and expected overtime costs. Let $X_{it} = 1$ if elective patient i is operated in period t for $t \in \{r_i, \ldots, H, H+1\}$, and $X_{it} = 0$ otherwise. The main problem can be formulated as follows:

$$\min_{\mathbf{X}} \sum_{i=1}^{N} \sum_{t=r_i}^{H+1} p_{it} X_{it} + \sum_{t=1}^{H} c_t \mathrm{E}_{W_t} \left[\left(W_t + \sum_{i \in I_t} d_i X_{it} - T_t \right)^{+} \right]$$

$$\text{s.t.} \sum_{t=r_i}^{H+1} X_{it} = 1, \quad \forall i \in \{1, 2, \ldots, N\}$$

$$X_{it} = \{0, 1\}, \quad \forall i \in \{1, 2, \ldots, N\}, t \in \{r_i, \ldots, H+1\}$$

where I_t is a set of patients assignable to period t, i.e., $I_t = \{i \in \{1, \ldots, N\}$ such that $r_i \leq t\}$,

$E_{W_t}[\cdot]$ is the expectation with respect to the probability distribution of W_t, and $(y)^+ = \max\{0, y\}$.

Note that the period $H+1$ is a"fictitious" period added to the planning horizon for patients that are rejected from the current planning and will be considered in the next horizon. Also the first constraint ensures that each elective patient is assigned exactly once.

Recommended Parameters Settings:

1. Number of periods $H = 5$ days;
2. Choose number of elective patients so that the expected capacity required for elective patients is (i) $\tau = 100$ percent and (ii) $\tau = 85$ percent of total capacity, i.e., choose (i) $N = 185$ and (ii) $N = 155$ ($\approx \dfrac{64 \text{ hours/day}}{1.75 \text{ hours/surgery}} \times 5$ days $\times$ .85);
3. Capacity $T_t = 8$ operating rooms $\times$ 8 hours $=$ 64 hours;
4. Overtime cost $c_t = \$500$ per hour for $t = \{1, \ldots, H\}$;
5. Total emergency capacity (need) W_t has an exponential distribution with mean $=$ 8 operating rooms $\times$ 1.2 hours $=$ 9.6 hours;
6. Release period $r_i = 1$, and duration of elective surgery $d_i \sim$ Uniform(0.5 hours, 3 hours) for $i = \{1, 2, \ldots, N\}$;
7. Cost of performing elective case i in period t is

$$\begin{aligned} p_{it} &= \$120 \times t, & t = r_i, \ldots, H; \\ &= \$120 \times (H+3), & t = H+1. \end{aligned}$$

Starting Solutions: Assign the N elective patients (roughly) equally across the H periods.
Measurement of Time: Number of generations of the random variable W_t.
Optimal Solutions: Unknown.
Known Structure: None.